NAME ________________________ **AP Calc BC HW 6.4, 6.5 Logistic Growth**

<u>Do not use your calculator.</u>

1. Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years.

A. Given $P(0) = 100$.

a. Find $\lim_{t \to \infty} P(t)$.

b. What is the range of the solution curve?

c. For what values of P is the solution curve increasing? Decreasing? Justify your answer.

d. For what values of P is the solution concave up? Concave down? Justify your answer.

e. Does the solution curve have an inflection point? Justify your answer.

f. Use the information you found to sketch the graph of $P(t)$.

B. Given $P(0) = 3000$.

a. Find $\lim_{t \to \infty} P(t)$.

b. What is the range of the solution curve?

c. For what values of P is the solution curve increasing? Decreasing? Justify your answer.

d. For what values of P is the solution concave up? Concave down? Justify your answer.

e. Does the solution curve have an inflection point? Justify your answer.

f. Use the information you found to sketch the graph of $P(t)$.

C. How many bears are in the park when the population of bears is growing the fastest?

2. Suppose a rumor is spreading through a dance at a rate modeled by the logistic differential equation $\frac{dP}{dt} = P\left(3 - \frac{P}{2000}\right)$. What is $\lim_{t \to \infty} P(t)$? What does this number represent in the context of this problem?

3. (From the 1998 BC Multiple Choice)

The population $P(t)$ of a species satisfies the logistic differential equation

$$\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right),$$

where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

4. Suppose a population of wolves grows according to the logistic differential equation $\frac{dP}{dt} = 3P - 0.01P^2$, where P is the number of wolves at time t in years. Which of the following statements are true?

I. $\lim_{t\to\infty} P(t) = 300$

II. The growth rate of the wolf population is greatest at $P = 150$.

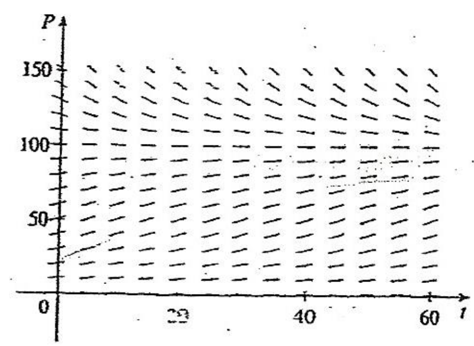
III. If $P > 300$, the population of wolves is increasing.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

5. Suppose that a population develops according to the logistic equation

$$\frac{dP}{dt} = 0.05P - 0.0005P^2$$

where t is measured in weeks.



(a) What is the carrying capacity?

(b) A slope field for this equation is shown at the right.

Where are the slopes close to 0?

Where are they largest?

Which solutions are increasing?

Which solutions are decreasing?

(c) Use the slope field to sketch solutions for initial populations of 20, 60, and 120.

What do these solutions have in common?

How do they differ?

Which solutions have inflection points?

At what population level do they occur?

Answers to Worksheet 1 on Logistic Growth

1. A a. 2500
   b. [100, 2500)
   c. increasing for [100, 2500)
   d. concave up for (100, 1250) and concave down for (1250, 2500)
   e. yes, IP when P = 1250
   f. sketch

   B. a. 2500
   b. (2500, 3000]
   c. decreasing for (2500, 3000]
   d. concave up for (2500, 3000]
   e. no
   f. sketch

   C. 1250
2. 6000; there are 6000 people at the dance.
3. E
4. C
5. (a) 100

   (b) Close to 0? $P = 0$ and $P = 100$
   Largest? $P = 50$
   Increasing? $P(0) < 100$
   Decreasing? $P(0) > 100$

   (c) In common? All have a limit of 100.
   Differ? Two are increasing; one is decreasing.
   Inflection points? The one with initial condition of 20.
   At what pop. level does the inflection point occur? When $P = 50$.